

RRB

Railway Recruitment Board

Junior Engineer

CBT 1

Computer Based Test - Stage 1

- **Mathematics**
- **General Intelligence & Reasoning**

Comprehensive Theory with
Solved Examples, Practice Sets & Previous Years Questions





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RRB-Junior Engineer : Mathematics and General Intelligence & Reasoning

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Preface

Railway Recruitment Board-Junior Engineer has always been preferred by Engineers due to job stability. Indian Railways is one of the biggest Government employers in India. With the exam being just a few months away, it is time for the candidates planning to appear for the exam to pull up their socks and start their RRB-JE preparation.



The RRB-JE exam is conducted in two stages as shown in table given below.

Papers	Subject	Maximum Marks	Duration
CBT-1 : Objective type	(i) Mathematics	30 Marks	90 Minutes
	(ii) General Intelligence and Reasoning	25 Marks	
	(iii) General Awareness	15 Marks	
	(iv) General Science	30 Marks	
	Total	100 Marks	
CBT-2 : Objective Type	(i) General Awareness	15 Marks	120 Minutes
	(ii) Physics and Chemistry	15 Marks	
	(iii) Basics of Computers and Applications	10 Marks	
	(iv) Basics of Environment and Pollution Control	10 Marks	
	(v) Technical Abilities (viz, CE, ME, EE, EC, CS etc)	100 Marks	
	Total	150 Marks	
Note: There shall be negative marking for incorrect answers in CBTs. Each question carries 1 mark and 1/3rd of the marks allotted for each question shall be deducted for each wrong answer. Candidates shortlisted in Stage 1 are called for Stage 2.			

MADE EASY has taken due care to provide complete solution with accuracy. Apart from Railway Recruitment Board-Junior Engineer, this book is also useful for Public Sector Examinations and other competitive examinations for engineering graduates.

I have true desire to serve student community by providing good source of study and quality guidance. I hope this book will prove as an important tool to succeed in RRB-JE and other competitive exams. Any suggestion from the readers for improvement of this book is most welcome.

With Best Wishes

B. Singh (Ex. IES)

CMD, MADE EASY

Exam Syllabus

(Computer Based Test 2019-First Stage)

Mathematics: Number systems, BODMAS, Decimals, Fractions, LCM and HCF, Ratio and Proportion, Percentages, Mensuration, Time and Work, Time and Distance, Simple and Compound Interest, Profit and Loss, Algebra, Geometry, Trigonometry, Elementary Statistics, Square Root, Age Calculations, Calendar & Clock, Pipes & Cistern.

General Intelligence and Reasoning: Analogies, Alphabetical and Number Series, Coding and Decoding, Mathematical operations, Relationships, Syllogism, Jumbling, Venn Diagram, Data Interpretation and Sufficiency, Conclusions and Decision Making, Similarities and Differences, Analytical reasoning, Classification, Directions, Statement – Arguments and Assumptions etc.

General Awareness: Knowledge of Current affairs, Indian geography, culture and history of India including freedom struggle, Indian Polity and constitution, Indian Economy, Environmental issues concerning India and the World, Sports, General scientific and technological developments etc.

General Science: Physics, Chemistry and Life Sciences (up to 10th Standard CBSE syllabus).



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Number Systems

CHAPTER

1



Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Digits and Numbers
- Arithmetical Operations
- Index form
- Arithmetic Progression
- Geometric Progression
- Important formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.



Introduction to the topic

With respect to preparation for the RRB-JE Examination, Number System has been seen to be one of the important topics in the Mathematics part of CBT-1 with a total of 3 questions in the last conducted exam. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of Number System questions asked in the RRB-JE.

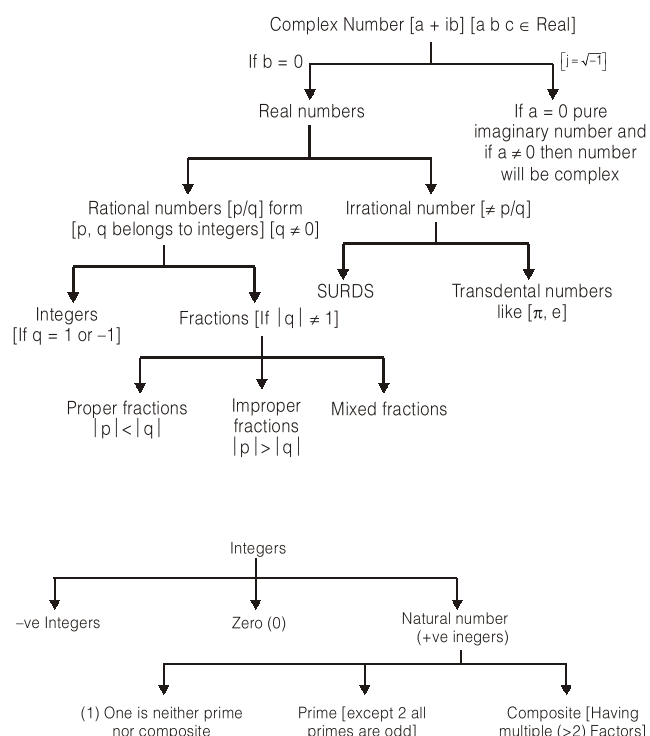


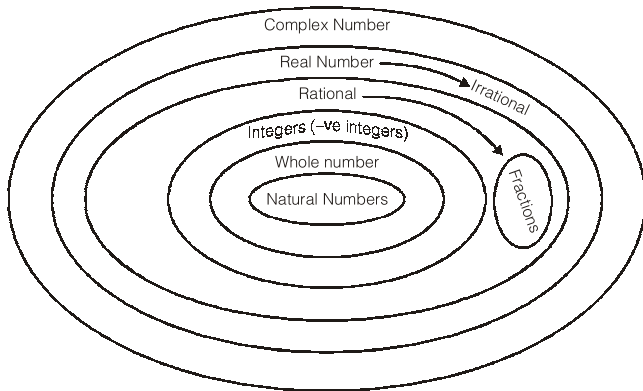
Type of Questions

Question asked from this topic are based on concepts: Digits and Numbers, Arithmetical Operations, Index form, Arithmetic Progression, Geometric Progression.

In Quantitative Aptitude (QA), Number System is one of the module which is of critical importance. We can consider this module as the back bone as well as basic foundation and building block for QA as well as for reasoning. Applications of concepts of numbers can be easily found in puzzles, reasoning based questions, number series and many more reasoning areas. This is why it is our suggestion to students to understand the concepts discussed in the module thoroughly alongwith understanding of applications.

1. Classifications of Numbers





Our main focus in this module of numbers is on **real number system**. However in context, of imaginary number only following property is important.

Imaginary Numbers

$$i = \sqrt{-1} \Rightarrow i^{4K+1} \equiv \sqrt{-1} \equiv i$$

$$i^2 = -1 \Rightarrow i^{4K+2} \equiv -1 \equiv i^2$$

$$i^3 = -i \Rightarrow i^{4K+3} \equiv -i \equiv i^3$$

$$i^4 = 1 \Rightarrow i^{4K} \equiv 1 \equiv i^4$$

Question: What is the value of expression

$$\frac{i^{12} + i^{13} + i^{14} + i^{15}}{i^{18} + i^{19} + i^{20} + i^{21}} ?$$

- (a) i^2 (b) -1
 (c) $1/i^2$ (d) None of these

Ans. (d)

Solution:

$$\frac{i^{12}(1+i+i^2+i^3)}{i^{18}(1+i+i^2+i^3)}$$

If we commit a mistake of cancelling out common terms in numerator and denominator options a, b, c all one correct hence my answer should be (d) but

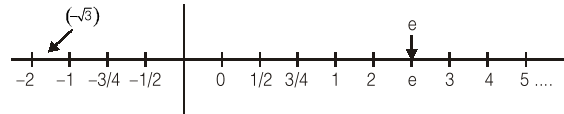
$$\begin{aligned} \text{expression } & 1 + i + i^2 + i^3 \\ & = 1 + i + (-1) + (-i) = 0 \end{aligned}$$

hence expression in question leading to undetermined

form $\left[\frac{0}{0} \right]$ hence correct answer is option (d).

Real Number System

Entire real numbers group of rational and irrational numbers combined forms the set of real number, which is represented by symbol $\rightarrow R$. All real numbers can be represented as points on a real number line.



Rational Number

All the numbers in p/q ($q \neq 0$) form are rational numbers [p, q are integers] set of rational number is represented by $\rightarrow Q$.

Rational Numbers Follow

Following forms of representations.

- (a) Terminating decimal forms

for example 0.125

$$\Rightarrow 0.125 = \frac{125}{1000} = \Rightarrow \text{Rational}$$

- (b) Nonterminating but recurring decimal forms.

- (i) For example

$$Q = 0.37373737 \dots$$

$$100Q = 37.373737 \dots$$

$$99Q = 37 \Rightarrow Q = 37/99 \Rightarrow \text{rational}$$

- (ii) For example

$$Q = 0.37292929 \dots$$

$$100Q = 37.292929 \dots$$

$$10000Q = 3729.292929 \dots$$

$$9900Q = (3729 - 37)$$

$$Q = \left(\frac{3729 - 37}{9900} \right)$$

$$= \frac{p}{q} \text{ form } \Rightarrow \text{rational}$$

Fraction

All rational numbers in which $|q| \neq 1$ comprise the set of fractions.

Proper Fractions

$$\text{If } |p| < |q|$$

then fraction is proper fraction. Value of proper fraction is always in between $(-1 \text{ to } +1)$

$$[-1 < p/q < 1]$$

Improper Fraction

$$\text{If } |p| > |q|$$

then fraction is improper fraction value of improper fraction is < -1 or more than $(>) 1$

Mixed Fraction

Just a modified form if improper fraction.

$$\text{Eg. } \frac{13}{4} \Rightarrow 3\frac{1}{4}$$

Improper fraction equivalent mixed fraction

$n * \text{LCM}(5, 6, 7) - 2 = n \times 210 - 2$
Highest possible three digit number will be 838.

Case-II

On dividing a number a, b and c if we get k as remainder always, then that number will be $(n - 1) \text{LCM of } (a, b, c) + k$.

Ex.21 On dividing a number by 5, 6 and 7 if we get 2 as remainder always, find that number

Sol. That number will be
 $(n - 1) \times \text{LCM of } [5, 6, 7] + 2$
 $\Rightarrow 2$ is such smallest number
next number will be $= 210 + 2 = 212$

Case-III

If a number after adding k is exactly divisible by a, b and c then that number will be.

$$n \times \text{LCM}(a, b, c) - k$$

Ex.22 Find a number which after adding 7 is divisible by 10, 11 and 12.

Sol. That number will be
 $n \times \text{LCM of } [10, 11, 12] - 7$
if $n = 1$ then
 $660 - 7 = 653$ Ans.

12. Base System

The Number system is used to represent any number using a set of symbols (digits /letters). The base defines the number of symbols in particular base system. We generally work in Decimal system as there are 10 digits (0, 1, 2,9). Some others systems are;

Binary base system: 2 symbols: 0, 1

Octal base system: 8 symbols: 0,1,2,3,4,5,6,7

Hexadecimal system: 16 symbols:

0,1,2,3,4,5,6,7,8,9, A = 10, B = 11, C = 12,

D = 13, E = 14, F = 15

Converting any number from any Base system to Decimal number system:

$$abcd.efg_B = a \times B^3 + b \times B^2 + c \times B^1 + d \times B^0 \\ + e \times B^{-1} + f \times B^{-2} + g \times B^{-3}$$

Example:

$$1234.56_8 = 1 \times 8^3 + 2 \times 8^2 + 3 \times 8^1 + 4 \times 8^0 \\ + 5 \times 8^{-1} + 6 \times 8^{-2} \\ = 512 + 128 + 24 + 4 + 0.625 + 0.093750 \\ = 668.718750$$

Converting any number from Decimal to other Base system:

Divide the number by base and get the first remainder r_1 and Quotient q_1 .

Now divided q_1 by base and get remainder r_2 and Quotient q_2 .

Repeat the following process till we get the quotient $q_n = 0$.

Now the decimal number in base b is $r_n r_{n-1} \dots r_3 r_2 r_1$.

Example 1:

1. $(149)_{10} = ()_7$

		Remainder
7	149	
7	21	2
7	3	0
	0	3

$$(149)_{10} = (302)_7$$

2. Add $(432)_7 + (355)_7$

11 ← carry

$$(432)_7$$

$$(355)_7$$

$$1120$$

$$\text{as } 2 + 5 = (7)_{10} = (10)_7$$

$$3 + 5 + 1 = (9)_{10} = (12)_7$$

$$1 + 4 + 3 = (8)_{10} = (11)_7$$

Solved Examples

Q.1 The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the the changed number.

- (a) 28 (b) 19
(c) 37 (d) 46

Sol. (a)

Going through options we get $82 - 28 = 54$.

Q.2 The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.

- (a) 11, 4 (b) 12, 3
(c) 13, 2 (d) 10, 5

Sol. (b)

Going through options only 12 and 3 satisfies the condition

$$AM = \frac{12+3}{2} = 7.5$$

$$GM = \sqrt{12 \times 3} = 6\sqrt{3} \text{ which is 20\% less than 7.5.}$$



Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Basic concept of Brackets
- Order
- Division and Multiplication
- Addition and Subtraction
- Algebraic formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.



Introduction to the topic

With respect to preparation for the RRB-JE Examination, BODMAS is an important topic in the Mathematics part of CBT-1 with a total of 1 question in the last conducted exam. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of BODMAS questions asked in the RRB-JE.

BODMAS, even though is a part of number systems yet, it has its own importance as far as this examination is concerned.



Type of Questions

Question asked from this topic are based on concepts: Basic concept of BODMAS, Brackets, Orders, Division, Multiplication, Addition, Subtraction and Algebraic formulae.

BODMAS is a useful acronym that lets you know which order to solve mathematical problems (or sums). It's important that you follow the rules of BODMAS as without it your answers can be wrong.

The BODMAS acronym is for

- Brackets (parts of a calculation inside brackets always come first).
- Orders (numbers involving powers or square roots).
- Division.
- Multiplication.
- Addition.
- Subtraction.

Brackets

Start with anything inside brackets, going from left to right.

Ex.1 $4 \times (3 + 2) = ?$

Sol. You need to do the operation, or sum, inside the brackets first, $3 + 2$, then multiply the answer by 4.

$$3 + 2 = 5$$

$$4 \times 5 = 20$$

If you ignored the brackets and did the sum $4 \times 3 + 2$ you would get 14. You can see how the brackets make a difference to the answer.

Orders

Do anything involving a power or a square root next (these are also known as orders), again working from left to right if there is more than one.

Ex.2 $3^2 + 5 = ?$

Sol. You need to do the power sum first, before you can add 5.

$$3^2 = 3 \times 3 = 9$$

$$9 + 5 = 14$$

Division and Multiplication

Once you have done any parts of the sum involving brackets or powers the next step is division and multiplication.

Multiplication and division rank equally, so you go from left to right in the sum, doing each operation in the order in which it appears.

Ex.3 $4 \times 5 \div 2 + 7 = ?$

Sol. You need to do division and multiplication first, but you have one of each.

Start from the left and work across to the right, which means that you start with $4 \times 5 = 20$. Then do the division, $20 \div 2 = 10$.

Only then do you move to the addition:

$$10 + 7 = 17. \text{ The answer is } 17.$$

Addition and Subtraction

The final step is to calculate any addition or subtraction. Again, subtraction and addition rank equally, and you simply move from left to right.

Ex.4 $4 + 6 - 7 + 3 = ?$

Sol. You simply start on the left and work your way across.

$$4 + 6 = 10$$

$$10 - 7 = 3$$

$$3 + 3 = 6$$

Solved Examples

Q.1 $(764 \times ?) \div 250 = 382$

- (a) 115 (b) 145
(c) 135 (d) 125

Sol. (d)

$$\frac{764 \times ?}{250} = 382$$

$$? = \frac{382 \times 250}{764} = 125$$

Q.2 $\frac{1}{4} \times (4856 \times 0.5) \times 12 = ?$

- (a) 7284 (b) 7462
(c) 7262 (d) 7414

Sol. (a)

$$\frac{1}{4} \times (4856 \times 0.5) \times 12 = 7284$$

Q.3 $853 + ? \div 17 = 1000$

- (a) 2516 (b) 2482
(c) 2499 (d) 16147

Sol. (c)

$$853 + \frac{?}{17} = 1000$$

$$\frac{?}{17} = 1000 - 853 = 147$$

$$? = 17 \times 147 = 2499$$

Q.4 $9643 - 7750 + ? = 4990$

- (a) 3079 (b) 3097
(c) 3090 (d) 4010

Sol. (b)

$$? = 4990 - 9643 + 7750 = 3097$$

Q.5 $6156 \div \sqrt{?} \times 53 = 4028$

- (a) 6889 (b) 6241
(c) 5929 (d) 6561

Sol. (d)

$$\frac{6153 \times 53}{\sqrt{?}} = 4028$$

$$\Rightarrow \sqrt{?} = 80.96 \approx 81 \Rightarrow ? = 6561$$

Previous RRB-JE Questions

Q.1 Find the value of $3 + \frac{1}{3 + \frac{2}{3 + \frac{4}{3}}}$

- (a) $\frac{148}{45}$ (b) $\frac{33}{14}$
(c) $\frac{38}{15}$ (d) $\frac{143}{30}$

[RRB (JE) 2014]

Sol. (a)

$$3 + \frac{1}{3 + \frac{2}{3 + \frac{4}{3}}} = 3 + \frac{1}{3 + \frac{2}{\frac{13}{3}}} = 3 + \frac{1}{3 + \frac{6}{13}} = 3 + \frac{13}{45} = \frac{148}{45}$$

Q.2 Solve the expression

$$\frac{(2+3) \times 5 + 3 \div \frac{1}{2}}{6 + 5 \times 4 \div \frac{4}{5}}$$



Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Operations on decimal fractions
- Conversion of vulgar fraction into decimal fraction
- Multiplication of a Decimal Fraction by a Power of 10.
- Comparison of fractions
- Recurring decimal - Pure and Mixed
- Division of decimal fraction by counting number
- Important formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.



Introduction to the topic

With respect to preparation for the RRB-JE Examination, Decimals is an important topic in the Mathematics part of CBT-1. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of Decimals questions asked in the RRB-JE.



Type of Questions

Question asked from this topic are based on concepts: Operations on decimal fractions, Conversion of vulgar fraction into decimal fraction, Comparison of fractions Recurring decimal - Pure and Mixed, Division of decimal fraction by counting number and Multiplication of a Decimal Fraction by a Power of 10.

1. Addition & Subtraction of Decimal Fractions

To add/subtract decimals,

- Write down the numbers, one under the other, with the decimal points lie in one column.
- Now the numbers can be added normally (remember to put the decimal point in the answer).

Examples:

- (a) $1.3 + 0.24 = ?$ (b) $1.25 + 0.1024 + 3 = ?$

1.3	1.25
0.24	0.1024
$\overline{1.54}$	3
	$\overline{4.3524}$

2. Multiplication of a Decimal Fraction by a Power of 10

Shift the decimal point to the right by as many places as the power of 10

For example:

- $3.78 \times 10 = 37.8$
- $0.0043 \times 100000 = 430$

3. Multiplication of Decimal Fractions

To multiply decimals,

- Multiply the given numbers considering them without decimal point.
- In the product obtained, place the decimal point by starting at the right and moving a number of places equal to the sum of the decimal places in the given numbers.

Examples:

- $4 \times .06 = ?$
 $4 \times 6 = 24$
Sum of the decimal places in the given numbers
 $= 0 + 2 = 2$



PRACTICE SET : QUESTIONS

- Q.1** $3889 + 12.952 - ? = 3854.002$
 (a) 47.095 (b) 47.752
 (c) 47.932 (d) 47.95
- Q.2** 0.04×0.0162 is equal to:
 (a) 6.48×10^{-3} (b) 6.48×10^{-4}
 (c) 6.48×10^{-5} (d) 6.48×10^{-6}
- Q.3** $\frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1}$ is equal to
 (a) 0.5 (b) 1.0
 (c) 20 (d) 22
- Q.4** The rational number for recurring decimal 0.125125 is
 (a) $\frac{63}{487}$ (b) $\frac{119}{993}$
 (c) $\frac{125}{999}$ (d) None of these
- Q.5** $0.002 \times 0.5 = ?$
 (a) 0.0001 (b) 0.001
 (c) 0.01 (d) 0.1
- Q.6** Which of the following is equal to 3.14×10^6 ?
 (a) 314 (b) 3140
 (c) 3140000 (d) None of these
- Q.7** $\frac{5 \times 1.6 - 2 \times 1.4}{1.3}$
 (a) 0.4 (b) 1.2
 (c) 1.4 (d) 4
- Q.8** How many digits will be there to the right of the decimal point in the product of 95.75 and .02554?
 (a) 5 (b) 6
 (c) 7 (d) None of these
- Q.9** $\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = ?$
 (a) 0.8 (b) 1.45
 (c) 2.40 (d) 3.25
- Q.10** 4.036 divided by 0.04 gives
 (a) 1.009 (b) 10.09
 (c) 100.9 (d) None of these

Solutions: Decimals

- 1. (d)**
 Let, $3889 + 12.952 - x = 3854.002$
 Then, $x = (3889 + 12.952) - 3854.002$
 $= 3901.952 - 3854.002 = 47.95$
- 2. (b)**
 $4 \times 162 = 648$. Sum of decimal places = 6.
 So, $0.04 \times 0.0162 = 0.000648 = 6.48 \times 10^{-4}$
- 3. (b)**
 Given expression = $\frac{(a^2 - b^2)}{(a + b)(a - b)} = \frac{(a^2 - b^2)}{(a^2 - b^2)} = 1$
- 4. (c)**
 $0.125125 \dots = 0.\overline{125} = \frac{125}{999}$
- 5. (b)**
 $2 \times 5 = 10$
 Sum of decimal places = 4
 $0.002 \times 0.5 = 0.001$
- 6. (c)**
 $3.14 \times 10^6 = 3.14 \times 1000000 = 3140000$.
- 7. (d)**
 Given expression = $\frac{8 - 2.8}{1.3} = \frac{5.2}{1.3} = \frac{52}{13} = 4$
- 8. (b)**
 Sum of decimal places = 7.
 Since the last digit to the extreme right will be zero (since $5 \times 4 = 20$), so there will be 6 significant digits to the right of the decimal point.
- 9. (a)**
 $\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = \frac{203 \times 292}{73 \times 145 \times 7} = \frac{4}{5} = 0.8$
- 10. (c)**
 $\frac{4.036}{0.04} = \frac{403.6}{4} = 100.9$





Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Proper and Improper fractions
- Mixed fractions
- Elements of a fractions
- Simplification of fractions
- Comparison of fractions
- Multiplication rules for two fractions
- Operation of fractions
- Important formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.



Introduction to the topic

With respect to preparation for the RRB-JE Examination, Fractions has been seen to be one of the important topics in the Mathematics part of CBT-1. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of Fractions asked in the RRB-JE.



Type of Questions

Question asked from this topic are based on concepts: Types of fractions, Conversion into decimals, Equating denominators, Operations of fractions - addition, subtraction, multiplication, division, Multiplication rules for two fractions, Elements of a fractions, Simplification of fractions and Comparison of fractions.

The three types of fractions are

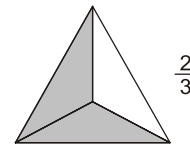
- Proper fraction
- Improper fraction
- Mixed fraction

1. Proper fraction

Fractions whose numerators are less than the denominators are called proper fractions. (Numerator < denominator)

For examples:

23, 34, 45, 56, 67, 29 58, 25, etc are proper fractions.



Two parts are shaded in the above diagram. Total number of equal parts is 3. Therefore, the shaded part can be represented as $\frac{2}{3}$ in fraction. The numerator (top number) is less compared to the denominator (bottom number). This type of fraction is called proper fraction.

Note: The value of a proper fraction is always less than 1.

2. Improper fraction

Fractions with the numerator either equal to or greater than the denominator are called improper fraction. (Numerator = denominator or, Numerator > denominator)

Fractions like $\frac{5}{4}$, $\frac{17}{5}$, $\frac{5}{2}$ etc. are not proper fractions.

These are improper fractions. The fraction $\frac{7}{7}$ is an improper fraction.

The fractions $\frac{5}{4}$, $\frac{3}{2}$, $\frac{8}{3}$, $\frac{6}{5}$, $\frac{10}{3}$, $\frac{13}{10}$, $\frac{15}{4}$, $\frac{9}{9}$, $\frac{20}{13}$, $\frac{13}{11}$, $\frac{14}{11}$, $\frac{17}{17}$ are the examples of improper fractions. The top number (numerator) is greater than the bottom number (denominator). Such type of fraction is called improper fraction.

A few final remarks

- Working with fractions does not modify the priority of operations.

For example:

$$\frac{2}{3} + \frac{4}{5} \times \frac{2}{3} = \frac{2}{3} + \frac{8}{15} = \frac{10}{15} + \frac{8}{15} = \frac{18}{15} = \frac{6}{5}$$

- A whole number can always be written as a fraction if an operation is to be done between the number and a fraction.

For example:

$$8 - \frac{5}{3} = \frac{8}{1} - \frac{5}{3} = \frac{24}{3} - \frac{5}{3} = \frac{19}{3}$$

- Avoid working with mixed numbers... transform them into simple fractions.

For example:

$$4\frac{2}{7} = 4 + \frac{2}{7} = \frac{28}{7} + \frac{2}{7} = \frac{30}{7}$$

Solved Examples

- Q.1** $2 \times ? - 6 = 676/26$ What will come in place of question mark?

- (a) 9 (b) 15/26
(c) 16 (d) 26

Sol. (c)

$$\begin{aligned} 2 \times x - 6 &= \frac{676}{26} \\ \therefore 2 \times x - 6 &= 26 \\ 2 \times x &= 32 \\ \therefore x &= 16 \end{aligned}$$

- Q.2** An integer is 10 more than its one-third part. The integer is

- (a) 15 (b) 12
(c) 18 (d) 25

Sol. (a)

$$\begin{aligned} \text{Let integer be } M \\ \Rightarrow M - \frac{M}{3} &= 10 \\ \therefore M &= 15 \end{aligned}$$

- Q.3** Convert $0.\overline{88}$ into vulgar fraction

- (a) 80/99 (b) 44/45
(c) 22/25 (d) 8/9

Sol. (d)

$$0.\overline{88} = \frac{88 - 8}{90} = \frac{8}{9}$$

- Q.4** Which of the following fraction is the smallest?

- (a) 12/14 (b) 13/19
(c) 17/21 (d) 7/8

Sol. (b)

$$\frac{12}{14} = 0.857 \quad ; \quad \frac{13}{19} = 0.684$$

$$\frac{17}{21} = 0.8095 \quad \text{and} \quad \frac{7}{8} = 0.875$$

Since 0.684 is the smallest, so 13/19 is the smallest fraction.

- Q.5** Which of the following fraction is greater than 2/3 and less than 4/5?

- (a) 1/2 (b) 9/10
(c) 3/4 (d) 5/6

Sol. (c)

$$\frac{1}{2} = 0.5 \quad ; \quad \frac{2}{3} = 0.66 \quad ; \quad \frac{4}{5} = 0.8 \quad ; \quad \frac{9}{10} = 0.9$$

$$\frac{3}{4} = 0.75, \quad \frac{5}{6} = 0.833,$$

Clearly, 0.8 lies between 0.75 and 0.833
3/4 lies between 2/3 and 4/5

**PRACTICE SET : QUESTIONS**

- Q.1** When Rs 250 added to 1/4th of a given amount of money makes it smaller than 1/3rd of the given amount of money by Rs 100. What is the given amount of money?

- (a) 350 (b) 600
(c) 4200 (d) 3600

- Q.2** A boy was asked to find the value of 7/12 of a sum of money. Instead of multiplying the sum by 7/12, he divided it by 7/12 and thus his answer exceeded the correct value by Rs.95. Find the correct value?

- (a) 95 (b) 49
(c) 84 (d) None of these

- Q.3** Eight people are planning to share equally the cost of a rental car, if one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by?

- (a) 3/2 (b) 7/8
(c) 5/7 (d) None of these



Learning Objectives

After completion of this chapter, you should have a thorough understanding of the following:

- Factor and Multiple
- Prime factorization method
- Division method
- Least Common Multiple
- LCM of power and base
- HCF of power and base
- HCF and LCM of Decimal numbers
- Important formulae
- Kinds of questions which are asked in RRB-JE
- Methods of solving.



Introduction to the topic

With respect to preparation for the RRB-JE Examination, LCM and HCF has been seen to be one of the important topics in the Mathematics part of CBT-1 with a total of 4 questions in the last conducted exam. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about different kinds of LCM and HCF asked in the RRB-JE.



Type of Questions

Question asked from this topic are based on concepts: Factor and Multiple, Prime factorization method, Division method, Least Common Multiple, HCF and LCM of Decimal numbers, LCM of power and base and HCF of power and base

Factor and Multiple

If one of the two numbers can divide the second number, then the first number is factor and the second number is multiple.

Least Common Multiple (LCM)

That least number which can be divided by the given numbers, is called least common multiple.

Method to find out LCM

1. Prime Factorization Method

Ex. 18, 28, 108 and 105. Find out the LCM.

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

$$28 = 2 \times 2 \times 7 = 2^2 \times 7$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$$

$$105 = 3 \times 5 \times 7$$

Now all the prime factors in their maximum power

$$2^2 \times 3^3 \times 5 \times 7 = 3780$$

2. Division Method

Ex. 36, 60, 84 and 90, Find the LCM.

2	36, 60, 84, 90
2	18, 30, 42, 45
3	9, 15, 21, 45
3	3, 5, 7, 15
5	1, 5, 7, 5
	1, 1, 7, 1

Now LCM = $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$

Types of LCM

1. LCM of Fractions

$$\text{LCM of fraction} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$$

Ex. $\frac{1}{3}, \frac{2}{9}$ and $\frac{5}{6}$, Find LCM

$$= \frac{\text{LCM of 1, 2 and 5}}{\text{HCF of 3, 9 and 6}} = \frac{10}{3}$$

2. LCM of Decimals

Eg. 2.4, 0.36 and 0.045, find out LCM

$$\begin{array}{r|l}
 3 & 24, 36, 45 \\
 \hline
 3 & 8, 12, 15 \\
 \hline
 2 & 8, 4, 5 \\
 \hline
 2 & 4, 2, 5 \\
 \hline
 & 2, 1, 5
 \end{array}$$

$$\text{LCM} = 3 \times 3 \times 2 \times 2 \times 2 \times 1 \times 5 = 360$$

$$\text{Hence the LCM} = 36.0 = 36$$

[**Note** : In such cases in 2.4 there is less number on the right to the decimal. Hence the decimal of such number will be put in the LCM or it can be found out by fraction method too]

3. LCM of power and Base

Type : 1

When base is same but power is different

Ex-1. $3^7, 3^{12}, 3^{17}$

$$\text{LCM} = 3^{17}$$

(Of maximum power/higher in value)

Ex-2. $5^{-9}, 5^{-7}, 5^{-17}$

$$\text{LCM} = 5^{-7} (\because -7 > -17)$$

Type : 2 When base and power both are different.

Ex.1 2^3 and 3^2 $\text{LCM} = 2^3 \times 3^2 = 72$

HCF (Highest Common Factor)

That highest number which can divide all the given numbers.

Methods to find HCF

1. Prime Factorization Method

Ex.1 28 and 32

$$28 = 2 \times 2 \times 7$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$\text{HCF} = 2 \times 2 = 4$$

(the common prime factorian)

2. Continued division method

Ex1. 493 and 928

$$\begin{array}{r}
 493 \overline{)928} (1 \\
 \underline{493} \\
 435 \overline{)493} (1 \\
 \underline{435} \\
 58 \overline{)435} (7 \\
 \underline{406} \\
 29 \overline{)58} (2 \\
 \underline{58} \\
 0
 \end{array}$$

$$\text{HCF} = 29.$$

Ex2. 828, 1311 and 1955

$$\begin{array}{r}
 828 \overline{)1311} (1 \\
 \underline{828} \\
 483 \overline{)828} (1 \\
 \underline{483} \\
 345 \overline{)483} (1 \\
 \underline{345} \\
 138 \overline{)345} (2 \\
 \underline{276} \\
 69 \overline{)138} (0 \\
 \underline{138} \\
 0
 \end{array}$$

Hence, HCF of 828 and 1311 = 69

Now,

$$\begin{array}{r}
 69 \overline{)1955} (28 \\
 \underline{1932} \\
 23 \overline{)69} (3 \\
 \underline{69} \\
 0
 \end{array}$$

The required HCF = 23

HCF of Fraction :

Formula :

$$\text{HCF of fraction} = \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$$

Ex. $\frac{16}{21}, \frac{8}{15}, \frac{2}{3}, \frac{24}{27}$

Here, $\frac{\text{HCF of } 16, 8, 2 \text{ and } 24}{\text{LCM of } 21, 15, 3 \text{ and } 27} = \frac{2}{1400} = \frac{1}{700}$

HCF of Decimals

Ex. 1.5, 0.24 and 0.036.

$$15 = 3 \times 5$$

$$24 = 3 \times 2 \times 2 \times 2$$

$$36 = 3 \times 3 \times 2 \times 2$$

HCF of the 15, 24 and 36 = 3

Hence the required HCF of the above decimal number = 0.003.

[**Note**: In 0.036, there is maximum number on the right to the decimal. Hence the decimal of such number will be put in the HCF. or It can be found by fraction method too].

HCF of power and base

Type 1 : When base is same

Ex.1 $2^8, 2^{10}, 2^{15}$

$$\text{HCF} = 2^8 \text{ (the least powered base)}$$